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$x^2 + y^2 = C$ and $x^2 + y^2 = D$ ($CD = B$) by the rule $x = \sqrt{B} \cos (\theta \mp \phi)$, $y = \sqrt{B} \sin (\theta \mp \phi)$. If we allow for the interchange of x and y and for negative values, the rule becomes

$$x = \sqrt{B} \cos (l\pi/2 \pm \theta \mp \phi), \quad y = \sqrt{B} \sin (l\pi/2 \pm \theta \mp \phi);$$

and on applying this method successively to A and its divisors we get the formula (2).

We wish to know how many distinct solutions are given by (2). Two solutions given by

$$\Theta' = k'\pi/2 + \sum_i r_i' \theta_i \quad \text{and} \quad \Theta'' = k''\pi/2 + \sum_i r_i'' \theta_i$$

are equal only if

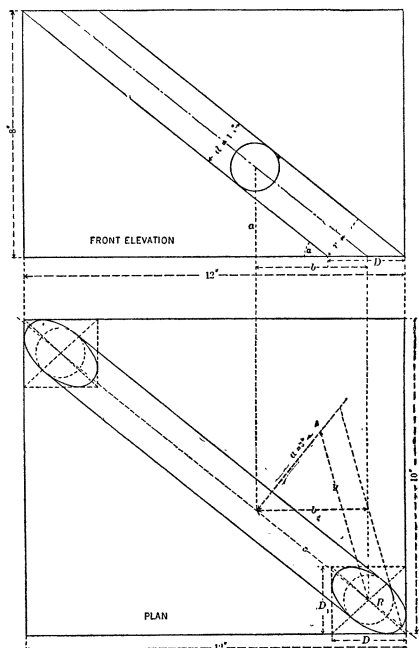
$$(k' - k'')\pi/2 + \sum_i (r_i' - r_i'')\theta_i$$

is a multiple of 2π . If this is so and $r_1' - r_1'' \neq 0$, it follows that

$$\cos (r_1' - r_1'')\theta_1 = \cos \left\{ K\pi/2 + \sum_{i=2}^m (r_i' - r_i'')\theta_i \right\}.$$

But the left member, being a polynomial of degree $|r_1' - r_1''|$ in $\cos \theta$ with integral coefficients of which the highest is a power of 2, is a fraction with denominator $a_i^{|r_1' - r_1''|}$, while the right member cannot be of this form. Hence $r_1' = r_1''$. Two solutions are then identical only if $r_i' = r_i''$ ($i = 1, \dots, m$) and $k' - k''$ is a multiple of 4. The number of effectively distinct combinations of values of k, r_1, \dots, r_n is therefore $4(n_1 + 1)(n_2 + 1) \dots (n_m + 1)$.

In some of the solutions given in (2), x and y will admit the common divisor $E = a_1^{s_1} a_2^{s_2} \dots a_m^{s_m}$. These are found by multiplying by E the roots of the equation $x^2 + y^2 = a_1^{n_1 - 2s_1} a_2^{n_2 - 2s_2} \dots$, and therefore occur in the cases where $|r_i| \leq n_i - 2s_i$ ($i = 1, 2, \dots, m$). The solutions have no common divisor when there is no number of the form E other than unity for which this condition holds, that is when $r_i = \pm n_i$ ($i = 1, \dots, m$). The number of relatively prime solutions is therefore $4 \cdot 2^m = 2^{m+2}$.



2696 [April, 1918]. Proposed by L. E. LUNN, Heron Lake, Minnesota.

An air pipe 18 inches in diameter passes diagonally through a room from one lower corner to the opposite upper corner leaving through elliptical openings in the floor and ceiling, so that the ellipses are tangent to two boundaries of the floor and to the two opposite boundaries of the ceiling. If the room is $10 \times 12 \times 8$ feet, find the remaining cubic capacity of the room.

SOLUTION BY A. R. NAUER, St. Louis, Missouri.

Make D the apparent width of the floor or ceiling contact as seen at the front elevation, and D_1 the same for the side elevation. Then

$$D = d/\sin \alpha = -\frac{27}{61.75} + \frac{\sqrt{468 \times 61.75 + 27^2}}{61.75} = 2.35025 +$$

and

$$D_1 = -\frac{22.5}{61.75} + \frac{\sqrt{369 \times 61.75 + 22.5^2}}{61.75} = 2.10715 +$$

a is the length of a perpendicular to the floor from an arbitrary point A on the center line of the pipe; a is here taken 3 feet.

r is the radius and d the diameter of the pipe.

R is the half major axis of the bounding ellipses, and

b is the distance, in front elevation from the foot of perpendicular a , to center of D , which is also the center of contact ellipses;

$$b = \frac{a}{8} (12 - D) = \frac{3}{8} (9.64975) = 3.61865, \quad b^2 = 13.095 +$$

c is the distance from the foot of perpendicular a to center of ellipse.

y is the distance from A to center of ellipse.

$$c^2 = b^2 \frac{(12 - D)^2 + (10 - D_1)^2}{(12 - D)^2} = 21.8552 +$$

$$y = \sqrt{a^2 + C^2} = \sqrt{30.8552} = 5.5547 +$$

$$R = \frac{r}{a} y = \frac{1}{4} y = 1.3887 +$$

Volume of pipe is $8\pi Rr = 2.51328 \times 1.3887 \times 0.75 = 26.17644 +$ cubic feet.

Hence remaining capacity of room is 960 cu. ft. $- 26.17644$ cu. ft. equal to 933.82356 cubic feet.

2722 [September, 1918]. Proposed by FRANK IRWIN, University of California.

The number of terms in the general polynomial of the n th degree in m variables and in that of the m th degree in n variables is the same. It would be interesting to devise schemes which, without assuming this result, should exhibit the terms of these polynomials in one-to-one correspondence with each other.

SOLUTION BY C. F. GUMMER, Queen's University.

Consider first the polynomial $P_n(x_1, x_2, \dots, x_m)$ of degree n , with coefficients all equal to 1. The general term is

$$x_1^{p_1} x_2^{p_2} \dots x_m^{p_m} x_{m+1}^{p_{m+1}}, \quad (\Sigma p = n),$$

where $x_{m+1} = 1$. Let the term be written at length, and a y ($= 1$) inserted after each group of like x 's except after the one for x_{m+1} , the y appearing even when the corresponding p is zero. The term is completely defined by the positions of the y 's, so that the subscripts may be dropped, and the term written $xx \dots yxx \dots yxx \dots$. Thus, in a polynomial of degree 4 in 5 variables, $x_1^2 x_2$ will be denoted by $xyxyyyxx$, the last x representing $x_5 = 1$. The various terms of P_n then correspond to the permutations of n x 's and m y 's. In the same way the terms of $P_m(x_1, x_2, \dots, x_n)$ of degree m may be made to correspond to the permutations of m x 's and n y 's. We may now put into one-to-one correspondence the terms of P_n and P_m which differ by interchange of the letters x and y .

Since the choice of a term in P_n corresponds to the choice of positions for the m y 's, this method furnishes a direct explanation of the fact that the number of combinations of $m + 1$ kinds of thing taking n things at a time and allowing repetition is $\binom{m+n}{n}$.

2729 [November, 1918]. Proposed by N. P. PANDYA, Sojitra, India.

Solve in integers $x^3 + 3y^4 = z^2$.

SOLUTION BY S. A. COREY, Des Moines, Iowa.

Having obtained by any means one solution x, y, z , it is easily seen that a^4x, a^3y, a^2z is a solution, where a may be any integer. Since 1, 2, 7 and 1, 1, 2 are solutions, $a^4, 2a^3, 7a^2$ and $a^4, a^3, 2a^2$ are solutions, whatever the value of a .

2731 [November, 1918]. Proposed by J. K. WHITEMORE, Yale University.

A bowl is in the form of a paraboloid of revolution. If for a given volume the surface is a minimum, prove that the ratio of the diameter of the top to the depth is approximately 1.86.

SOLUTION BY ELIJAH SWIFT, University of Vermont.

Let the parabola have the equation, $y = kx^2$. Call the depth of the bowl, l . The required ratio is